PRINCETON UNIVERSITY
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
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September 22, 1965.

Case File

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Gentlemen:

Enclosed are copies of Special Reports #3, 4, 5, 6, 7, 8 and 9 prepared by Mr. C. Ernesto S. Lindgren, Visiting Research Engineer to our department:

<u>Four-Dimensional Descriptive Geometry Rotation - Descriptive Solution</u>, March 1965.

<u>Four-Dimensional Descriptive Geometry Problems on 3-D Spaces</u>, March 1965.

<u>Graphical Plotting of Data. An Application of Three and Four Dimensional Theoretical Descriptive Geometries</u>, March 1965.

<u>Four-Dimensional Descriptive Geometry Metric Problems: Angles - Descriptive Solution</u>, May 1965.

<u>Four-Dimensional Descriptive Geometry Proposition of a Problem</u>, June 1965.

<u>Four-Dimensional Descriptive Geometry Metric Problems: Distances</u>, June 1965.

<u>Four-Dimensional Descriptive Geometry Proposed Problems</u>, July 1965.

These reports have been published as part of our continuing Engineering Graphics Technical Seminar Series.

I would appreciate it if you would consider including these reports in your monthly bibliographic index.

Sincerely yours,

Steve M. Slaby

Steve M. Slaby, Chairman
Department of Graphics and
Engineering Drawing.

SMS:sf
Enclosures.

ENGINEERING GRAPHICS SEMINAR

# FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY
# PROPOSITION OF A PROBLEM

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June 1965



TECHNICAL SEMINAR SERIES

Special Report No. 7

Department of Graphics and Engineering Drawing
School of Engineering and Applied Science
Princeton University

## ABSTRACT

The paper proposes the determination of the coordinates of the point of a plane, in a four-dimensional space, equidistant to three 3-D spaces of the system of reference for the Four-Dimensional Descriptive Geometry.

## GRAPHICAL DETERMINATION OF THE POINT

Given a plane by three of its points, (a), (b), and (c), Figure 1 shows the graphical determination of the point (t) of the plane equidistant to three 3-D spaces of the system of reference[1].

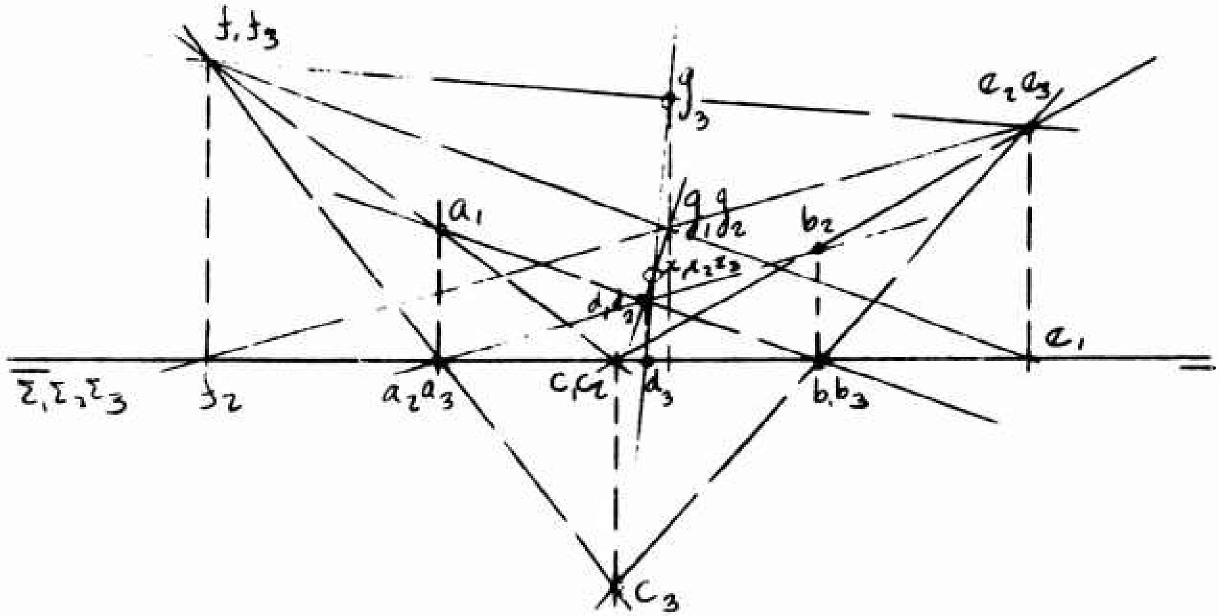


Figure 1

[1] See also the author's "Descriptive Geometry of Four Dimensions", Engineering Graphics Seminar, Technical Seminar Series, Report No. 9, Department of Graphics and Engineering Drawing, Princeton University, December 5, 1963, pp. 31-35.

To generalize the proposition, let (a), (b), and (c) be given as shown in Figure 2. Again (x) is the point of the plane, equidistant to $\Sigma_1$, $\Sigma_2$, $\Sigma_3$.

Let us now identify three complete plane quadrangles, by combining the projections of the lines (ab), (bc), and (ac), in pairs. See Figures 3, 4, and 5, noticing that we notate common points on the reference line, by the same letters.

The problem is proposed as follows:

1) Determine the coordinates of the point (x) with projections, $x_1$, $x_2$, $x_3$ in function of the anharmonic ratio of the three involutions (on the reference line) of figures 3, 4, and 5.

2) In determining these coordinates, determine also the equations of the lines ($0_1 0_2$ - $0_1' 0_2'$) - Figure 3, ($s_1 s_3$ - $s_1' s_3'$) - Figure 4, ($1_2 1_3$ - $1_2' 1_3'$) - Figure 5, in function of the corresponding anharmonic ratios.

3) Calling

$R_1$ - the anharmonic ratio for the involution, on the reference line, in Figure 3.

$R_2$ - the anharmonic ratio for the involution, on the reference line, in Figure 4

$R_3$ - the anharmonic ratio for the involution, on the reference line, in Figure 5.

check if:

$$\varphi_1 = \frac{R_1 R_3}{R_2} = (m'm''nn')$$

$$\varphi_2 = R_1 R_2 R_3 = (n'n''mm')$$

4) Derive a relation among the three anharmonic ratios, indicating that the point (x) is unique.

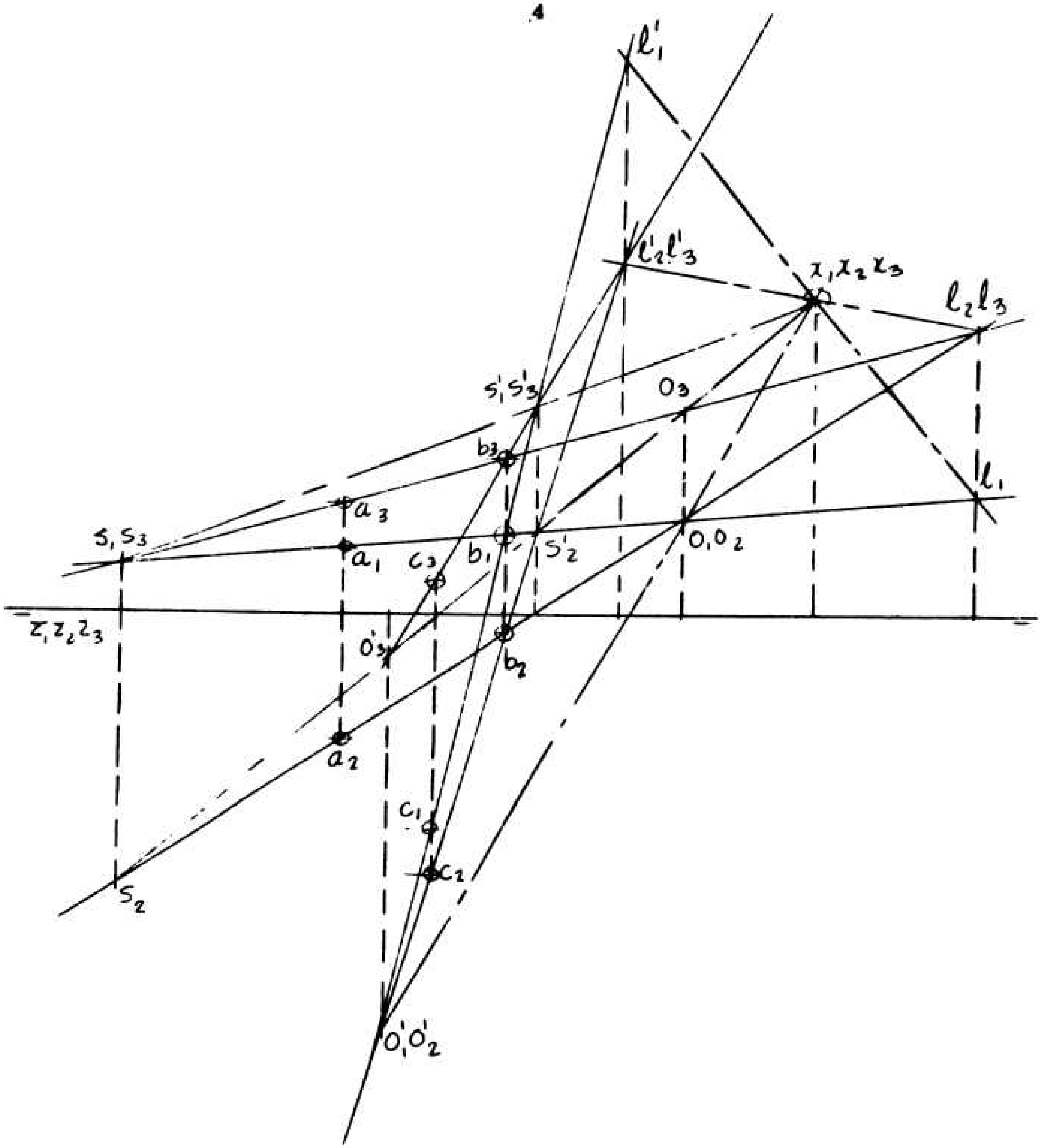
$l'_1$
$l'_2 l'_3$
$x_1 x_2 x_3$
$l_2 l_3$
$s'_1 s'_3$
$o_3$
$b_3$
$a_3$
$l_1$
$s_1 s_3$
$a_1$
$c_3$
$b_1$
$s'_2$
$o_1 o_2$
$z_1 z_2 z_3$
$o'_3$
$b_2$
$a_2$
$c_1$
$c_2$
$s_2$
$o'_1 o'_2$

**Figure 2**

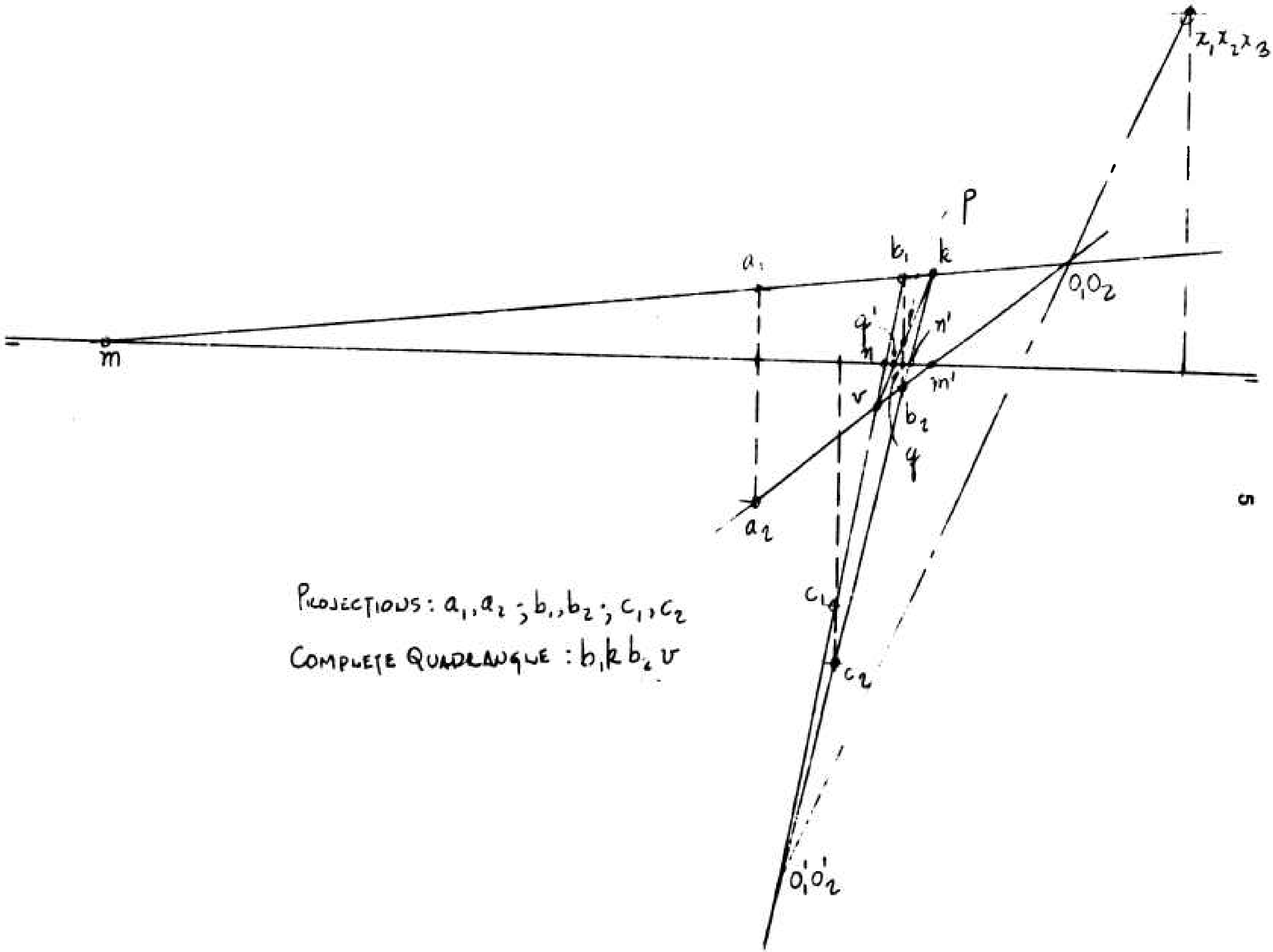
$x_1 x_2 x_3$
$p$
$a_1$
$b_1$
$k$
$O_1 O_2$
$q'$
$n'$
$m$
$n$
$m'$
$v$
$b_2$
$q$
$a_2$
$c_1$
$c_2$
$O_1' O_2'$
Projections: $a_1, a_2$; $b_1, b_2$; $c_1, c_2$
Complete Quadrangue : $b_1 k b_2 v$

**Figure 3**

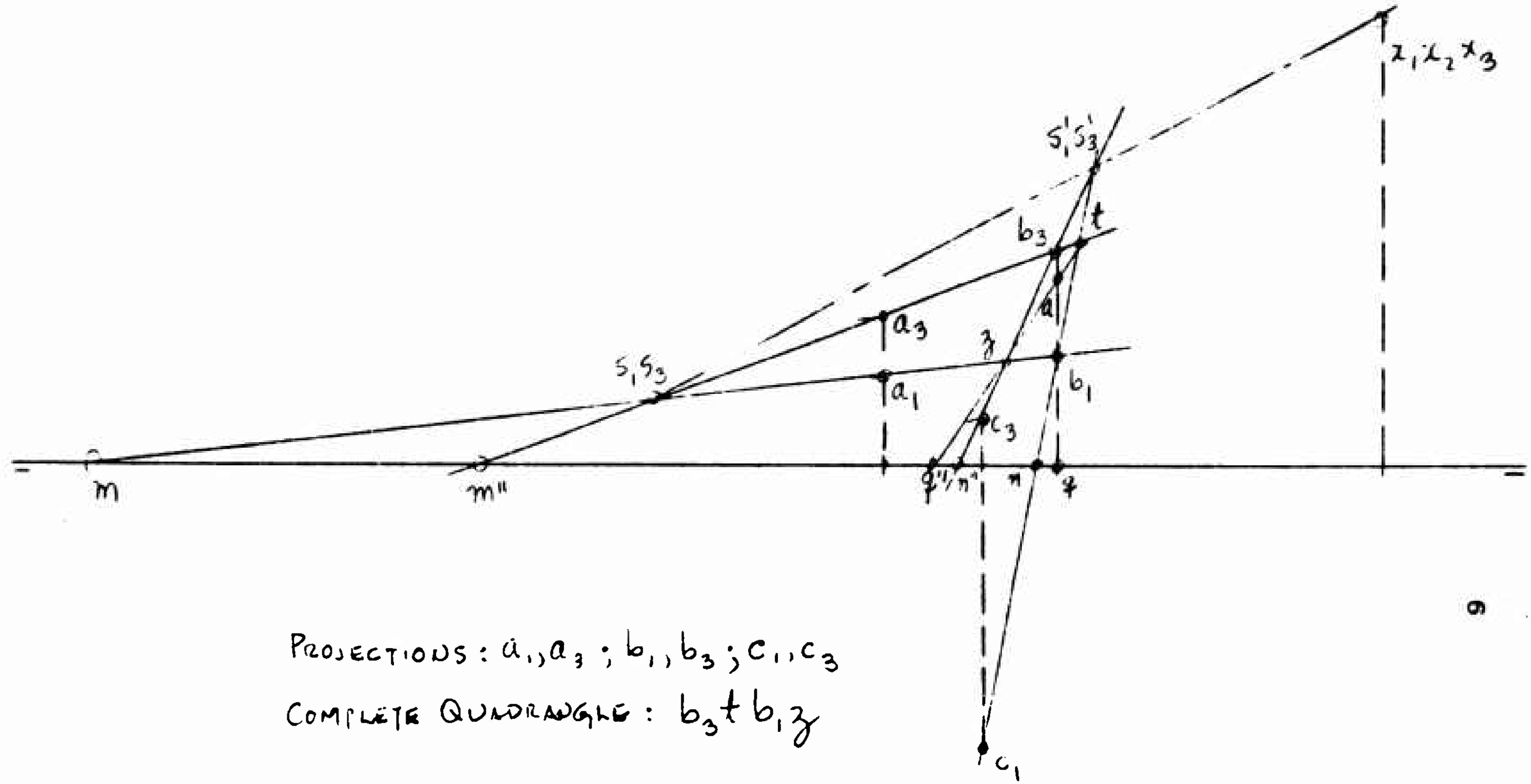
$x_1 x_2 x_3$
$s_1' s_3'$
$t$
$b_3$
$a_3$
$z$
$b_1$
$s_1 s_3$
$a_1$
$c_3$
$m$
$m''$
$c_1$
PROJECTIONS : $a_1, a_3$ ; $b_1, b_3$ ; $c_1, c_3$
COMPLETE QUADRANGLE : $b_3 t b_1 z$

**Figure 4**

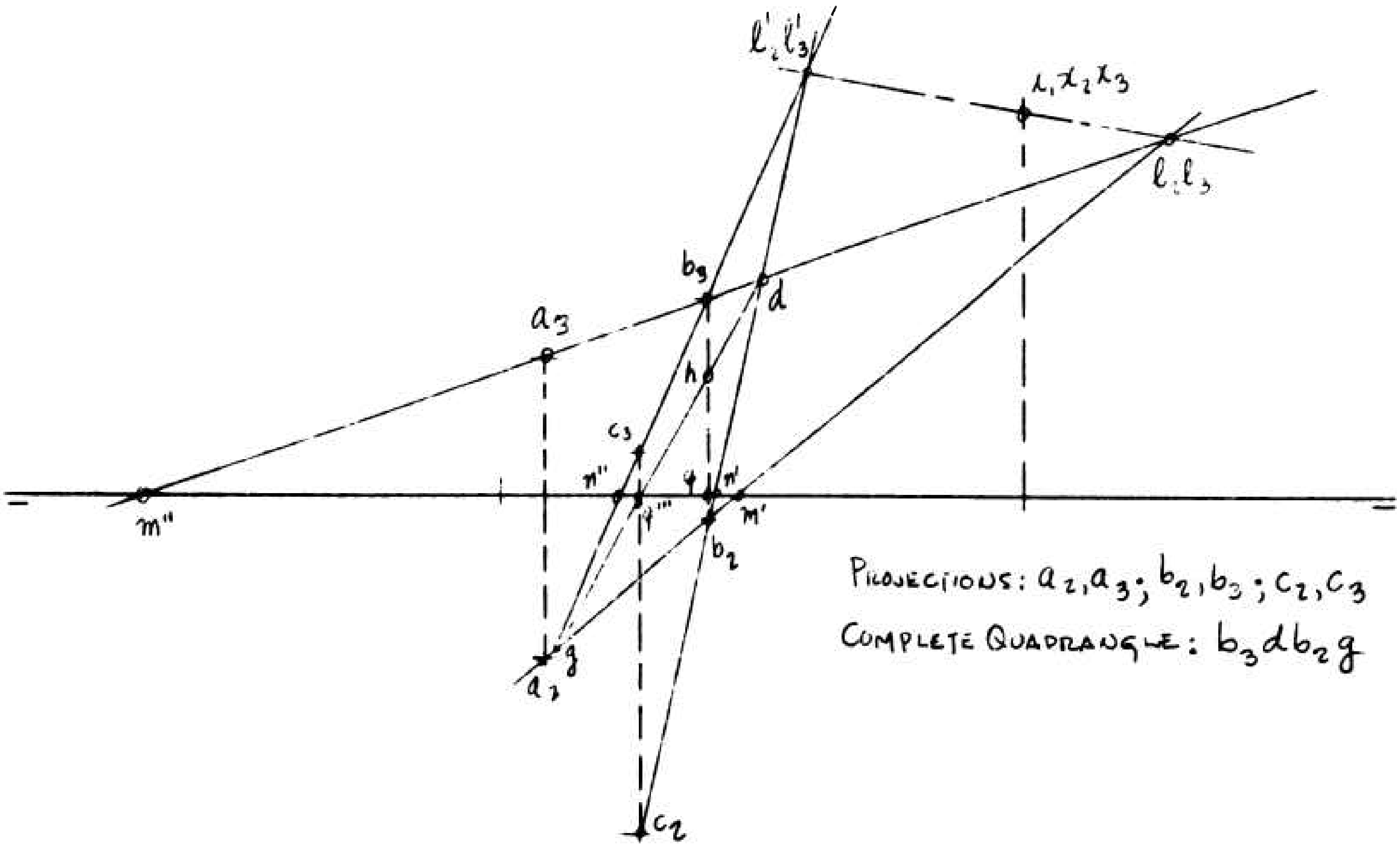
l'₁l'₃
l₁x₂x₃
l₁l₃
b₃
d
a₃
h
c₃
m''
φ
m'
φ'''
m'
b₂
g
a₂
c₂
m''
PROJECTIONS: a₂, a₃; b₂, b₃; c₂, c₃
COMPLETE QUADRANGLE: b₃db₂g

**Figure 5**